

# Light Scalar Mesons in the QCD Sum Rule

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We study the light scalar mesons in the QCD sum rule. We find that there are five independent scalar tetraquark currents in the local form, and we perform QCD sum rule analysis using both these currents and their combinations. Compared with the sum rules by conventional  $\bar{q}q$  currents, our result supports a tetraquark structure for low-lying scalar mesons.

## §1. Introduction

The nature of light scalar mesons of *up*, *down* and *strange* quarks is not fully understood.<sup>1),2)</sup> The expected members are  $\sigma(600)$ ,  $\kappa(800)$ ,  $f_0(980)$  and  $a_0(980)$  forming a nonet of flavor SU(3). Because they have the same spin and parity as the vacuum,  $J^P = 0^+$ , they reflect the bulk properties of the non-perturbative QCD vacuum. So far, several different pictures for the scalar mesons have been proposed. In the conventional quark model, they have a  $\bar{q}q$  configuration of  $^3P_0$  whose masses are expected to be larger than 1 GeV due to the  $p$ -wave orbital excitation. Furthermore, the mass ordering in a naive quark mass counting of  $m_u \sim m_d < m_s$  implies  $m_\sigma \sim m_{a_0} < m_\kappa < m_{f_0}$ . In chiral models, they are regarded as chiral partners of the Nambu-Goldstone bosons ( $\pi, K, \eta, \eta'$ ).<sup>3)</sup> Due to the collective nature, their masses are expected to be lower than those of the quark model. Yet another interesting picture is that they are tetraquark states.<sup>4)-7)</sup> In contrast with the  $\bar{q}q$  states, their masses are expected to be around 0.6 – 1 GeV with the ordering of  $m_\sigma < m_\kappa < m_{f_0, a_0}$ , consistent with the recent experimental observation.<sup>1),2),8)</sup> If such tetraquarks survive, they may be added to members of exotic multi-quark states.

In this contribution, we would like to report the results of a systematic study of the masses of the tetraquark scalar mesons in the QCD sum rule. We find that the QCD sum rule analysis with tetraquark currents implies the masses of scalar mesons in the region of 600 – 1000 MeV with the ordering,  $m_\sigma < m_\kappa < m_{f_0, a_0}$ , while the conventional  $\bar{q}q$  currents imply masses around 1.5 GeV.

## §2. Independent Currents

Let us start with currents for the scalar tetraquark, which we consider only local currents. Using the antisymmetric combination for diquark flavor structure,

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we arrive at the following five independent currents<sup>9)</sup>

$$\begin{aligned}
S_3^\sigma &= (u_a^T C \gamma_5 d_b)(\bar{u}_a \gamma_5 C \bar{d}_b^T - \bar{u}_b \gamma_5 C \bar{d}_a^T), \\
V_3^\sigma &= (u_a^T C \gamma_\mu \gamma_5 d_b)(\bar{u}_a \gamma^\mu \gamma_5 C \bar{d}_b^T - \bar{u}_b \gamma^\mu \gamma_5 C \bar{d}_a^T), \\
T_6^\sigma &= (u_a^T C \sigma_{\mu\nu} d_b)(\bar{u}_a \sigma^{\mu\nu} C \bar{d}_b^T + \bar{u}_b \sigma^{\mu\nu} C \bar{d}_a^T), \\
A_6^\sigma &= (u_a^T C \gamma_\mu d_b)(\bar{u}_a \gamma^\mu C \bar{d}_b^T + \bar{u}_b \gamma^\mu C \bar{d}_a^T), \\
P_3^\sigma &= (u_a^T C d_b)(\bar{u}_a C \bar{d}_b^T - \bar{u}_b C \bar{d}_a^T),
\end{aligned} \tag{2.1}$$

where the sum over repeated indices ( $\mu, \nu, \dots$  for Dirac, and  $a, b, \dots$  for color indices) is taken. Either plus or minus sign in the second parentheses ensures that the diquarks form the antisymmetric combination in the flavor space. The currents  $S$ ,  $V$ ,  $T$ ,  $A$  and  $P$  are constructed by scalar, vector, tensor, axial-vector, pseudoscalar diquark and antidiquark fields, respectively. The subscripts 3 and 6 show that the diquarks (antidiquark) are combined into the color representation  $\bar{\mathbf{3}}_c$  and  $\mathbf{6}_c$  ( $\mathbf{3}_c$  or  $\bar{\mathbf{6}}_c$ ), respectively. The currents for other members are formed similarly. We can also use a symmetric combination for diquark flavor structure. However, they are related to the antisymmetric ones by the axial  $U(1)$  transformation.<sup>10)</sup>

### §3. QCD Sum Rule Analysis

For the past decades QCD sum rule has proven to be a very powerful and successful non-perturbative method.<sup>11), 12)</sup> In sum rule analyses, we consider two-point correlation functions:

$$\Pi(q^2) \equiv i \int d^4x e^{iqx} \langle 0 | T \eta(x) \eta^\dagger(0) | 0 \rangle, \tag{3.1}$$

where  $\eta$  is an interpolating current for the tetraquark. We compute  $\Pi(q^2)$  in the operator product expansion (OPE) of QCD up to certain order in the expansion, which is then matched with a hadronic parametrization to extract information of hadron properties. At the hadron level, we express the correlation function in the form of the dispersion relation with a spectral function:

$$\Pi(p) = \int_0^\infty \frac{\rho(s)}{s - p^2 - i\varepsilon} ds, \tag{3.2}$$

where

$$\begin{aligned}
\rho(s) &\equiv \sum_n \delta(s - M_n^2) \langle 0 | \eta | n \rangle \langle n | \eta^\dagger | 0 \rangle \\
&= f_X^2 \delta(s - M_X^2) + \text{higher states}.
\end{aligned} \tag{3.3}$$

For the second equation, as usual, we adopt a parametrization of one pole dominance for the ground state  $X$  and a continuum contribution. The mass of the state  $X$  can be obtained

$$M_X^2 = \frac{\int_0^{s_0} e^{-s/M_B^2} s \rho(s) ds}{\int_0^{s_0} e^{-s/M_B^2} \rho(s) ds}. \tag{3.4}$$

We performed the sum rule analysis using all currents and their various linear combinations, and found a good sum rule by a linear combination of  $A_6^\sigma$  and  $V_3^\sigma$

$$\eta_1^\sigma = \cos \theta A_6^\sigma + \sin \theta V_3^\sigma, \quad (3.5)$$

where the best choice of the mixing angle turns out to be  $\cot \theta = 1/\sqrt{2}$ . For  $\kappa$ ,  $f_0$  and  $a_0$ , we have also found that similar linear combinations give better sum rules. The results of OPE can be found in Ref.<sup>16)</sup>

#### §4. Numerical Analysis

For numerical calculations, we use the following values of condensates:<sup>13)–15)</sup>  $\langle \bar{q}q \rangle = -(0.240 \text{ GeV})^3$ ,  $\langle \bar{s}s \rangle = -(0.8 \pm 0.1) \times (0.240 \text{ GeV})^3$ ,  $\langle g_s^2 GG \rangle = (0.48 \pm 0.14) \text{ GeV}^4$ ,  $m_u = 5.3 \text{ MeV}$ ,  $m_d = 9.4 \text{ MeV}$ ,  $m_s(1 \text{ GeV}) = 125 \pm 20 \text{ MeV}$ ,  $\langle g_s \bar{q}\sigma Gq \rangle = -M_0^2 \times \langle \bar{q}q \rangle$ ,  $M_0^2 = (0.8 \pm 0.2) \text{ GeV}^2$ .

The sum rules are written as power series of the Borel mass  $M_B$ . Since the Borel transformation suppresses the contributions from  $s > M_B$ , smaller values are preferred to suppress the continuum contributions also. However, for smaller  $M_B$  convergence of the OPE becomes worse. Therefore, we should find an optimal  $M_B$  preferably in a small value region. We have found that the minima of such a region are 0.4 GeV for  $\sigma$ , 0.5 GeV for  $\kappa$  and 0.8 GeV for  $f_0$  and  $a_0$ , where the pole contributions reach around 50 % for all cases.<sup>16)</sup> As  $M_B$  is increased, the pole contributions decrease, but the resulting tetraquark masses are stable as shown in Fig. 1.

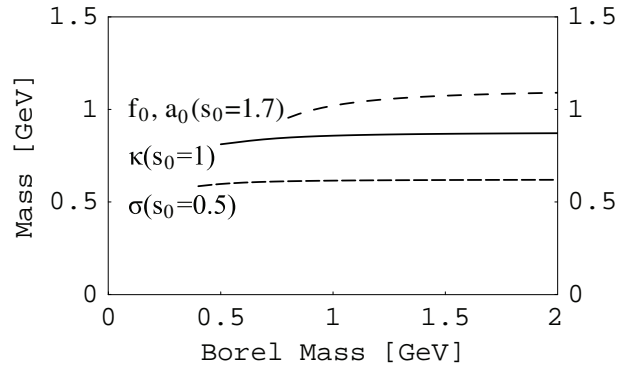


Fig. 1. Masses of the  $\sigma$  (short-dashed),  $\kappa$  (solid),  $f_0$  and  $a_0$  (long-dashed) mesons calculated by the tetraquark currents as functions of the Borel mass  $M_B$ , with  $s_0$  ( $\text{GeV}^2$ ) as shown in figures.

After careful test of the sum rule for a wide range of parameter values of  $M_B$  and  $s_0$ , we have found reliable sum rules, with which we find the masses  $m_\sigma = (0.6 \pm 0.1) \text{ GeV}$ ,  $m_\kappa = (0.8 \pm 0.1) \text{ GeV}$ ,  $m_{f_0, a_0} = (1 \pm 0.1) \text{ GeV}$ , which are consistent with the experimental results.<sup>2)</sup>

For comparison, we have also performed the QCD sum rule analysis using the  $\bar{q}q$  current within the present framework. The stable (weak  $M_B$ ) behavior is obtained with the masses of all four mesons around 1.5 GeV. Here again we have tested various values of  $M_B$  and  $s_0$ , and confirmed that the result shown is optimal.

## §5. Conclusions

We have performed the QCD sum rule analysis with tetraquark currents, which implies the masses of scalar mesons in the region of 600 – 1000 MeV with the ordering,  $m_\sigma < m_\kappa < m_{f_0, a_0}$ . We have also performed the QCD sum rule analysis with the conventional  $\bar{q}q$  currents, which implies masses around 1.5 GeV. We have tested all possible independent tetraquark currents as well as their linear combinations. Our observation supports a tetraquark structure for low-lying scalar mesons. To test the validity of the tetraquark structure, it is also important to study decay properties, which is often sensitive to the structure of wave functions. Such a tetraquark structure will open an alternative path toward the understanding exotic multiquark dynamics which one does not experience in the conventional hadrons.

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